Electromagnetic

Lecture (7)

مركز الفجر
للخدمات العلمية
٠١٠٠٩٢٦٤٠٠٦

Chapter (4)

"Currents and conductors"

2nd year

communications

Lecture (7) :

Chapter (4)

Currents and Conductors

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[I] Current and Current density :

ⓐ Electric Current I : (A)

هو المعدل الزمني لانتقال الشحنة الكهربية خلال مقطع معين من الموصل.

$$\Rightarrow I = \frac{dQ}{dt}$$

$$I = \frac{dQ}{dt}$$

$$I = \int_S \bar{J} \cdot ds = \bar{J} \cdot \bar{S}$$

ⓑ Current density : $\bar{J}$ (A/m^2)

هو مقدار التيار الكهربي الذي يمر عموديا على وحدة المساحات من مقطع الموصل.

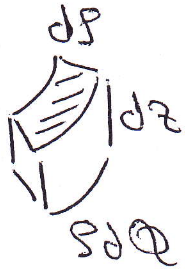
$$\Rightarrow I = \int_S \bar{J} \cdot d\bar{S}$$

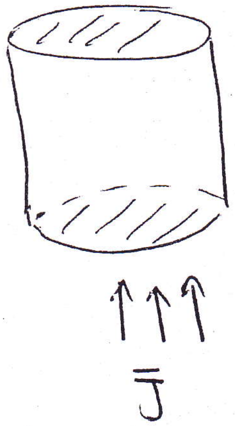
$$= \bar{J} \cdot \bar{S} \qquad (\text{if } \bar{J} = \text{Const})$$

Sheet (4)

$\boxed{1}$ $\bar{J} = 15(1-e^{-1000\rho})\,\hat{z}$

$d\bar{S} = \rho\, d\rho\, d\phi\, \hat{z}$





$$I = \int_S \bar{J}\cdot d\bar{S}$$

$$= \int_0^{2\pi}\int_0^{0.002} 15(1-e^{-1000\rho})\,\rho\, d\rho\, d\phi$$

$= 0.1325$ mA (using Calculator)

$\boxed{2}$ Conductor Properties and B.Cs:

⇒ Conductor Properties:

(a) Inside the Conductor:

$Q = 0 \longrightarrow \rho_v = 0$

$\bar{D} = 0 \longrightarrow \bar{E} = 0$

. الشحنات الكهربية بداخل الموصل تكون حرة الحركة

وبالتالى إذا تم شحن أى موصل بشحنة فإن هذه

الشحنة تتركز على سطح الموصل (وتكون محصلة الشحنة

داخل الموصل = صفر) .

(b) The Conductor Charge is distributed over its surface $\Rightarrow \rho_s$.

(c) The Conductor surface is an equipotential surface.

(d) The $\bar{E}$ at the surface of a Conductor is directed normal to its surface ($\perp$ surface) $\Rightarrow$ BCs.

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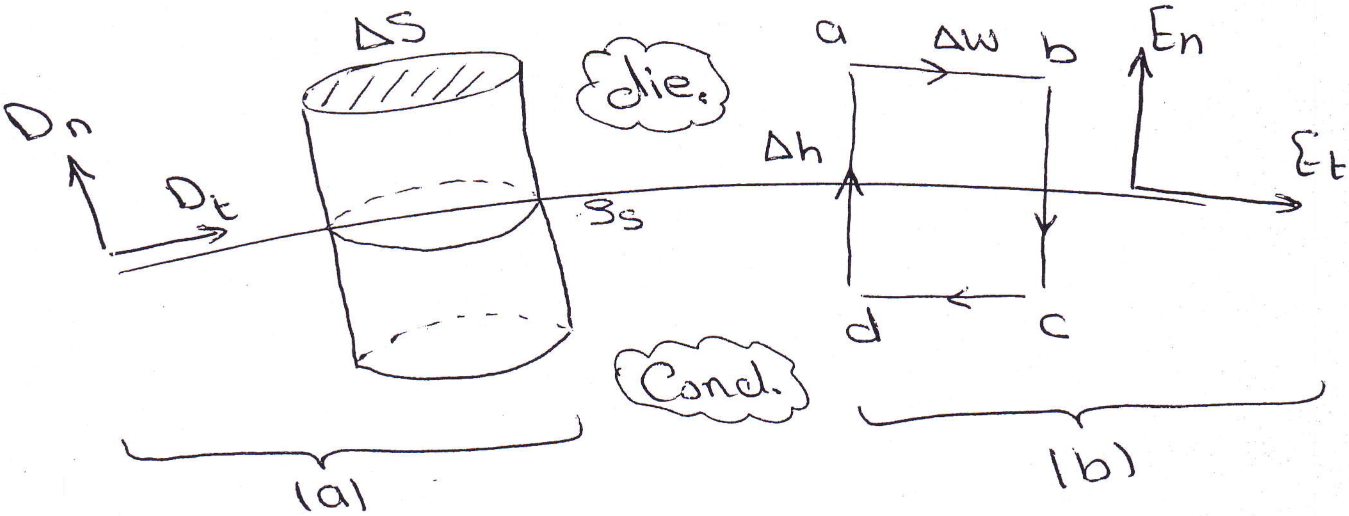
⇛ Boundary Conditions :



(a) Gauss law : $Q_{enc} = \bar{D} \cdot \bar{S}$

سطح جاوس تم اختياره على انه إسطوانة كما بالشكل

$Q_{enc} = \rho_s \, \Delta S$

$Q_{enc} = D_n \cancel{\Delta S} = \rho_s \cdot \cancel{\Delta S}$

$D_n = \rho_s$

$\bar{D} \cdot \bar{S} = D_n \, \Delta S$

$\therefore \; D_n \, \Delta S = \rho_s \, \Delta S$

$$\boxed{D_n = \rho_s}$$

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$\oint E \cdot dl =$

(b) $\oint_L \bar{E} \cdot d\bar{L} = 0$

المسار المغلق تم اختياره على أنه ($\overrightarrow{abcda}$) :.

$E_t \Delta w - \frac{1}{2} \cancel{\Delta h E_n} + 0 + \frac{1}{2} \cancel{\Delta h E_n} = 0$

حذف السفلي
لان بالموصل
E = 0

$$\therefore \boxed{E_t = 0}$$

$E_t = 0$

$\therefore$ The BCs of a Conductor :

(1) $E_t = 0$.

(2) $D_n = \rho_s$ (if $\rho_s \rightarrow$ not given $\therefore \rho_s = 0$)
"$D_n = 0$"

Sheet (5)

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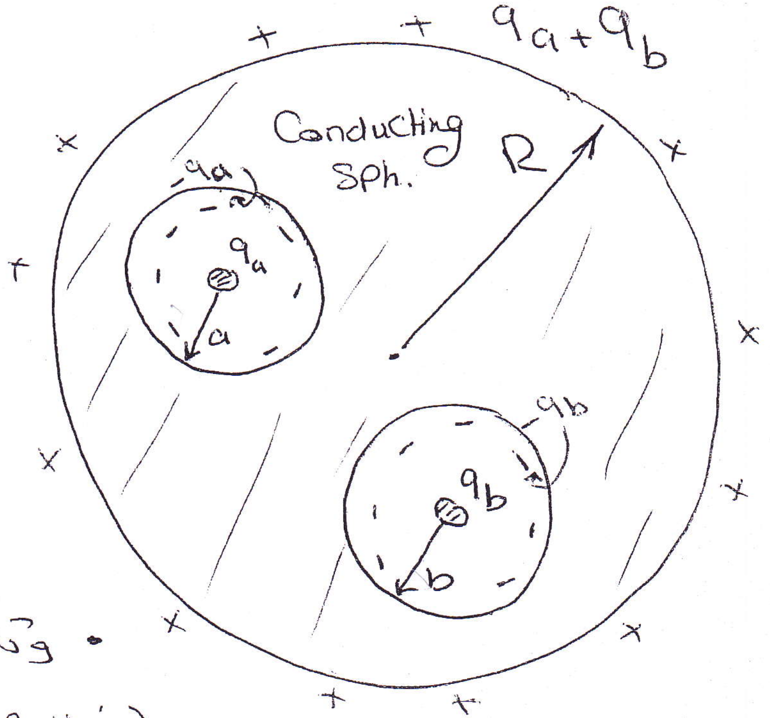


• نتيجة وجود الشحنتين q_a و q_b بداخل الفجوتين تنشأ شحنة تأثيرية (مستحثة) على السطح الداخلي للفجوتين. $\left(\begin{matrix}-q_a\\-q_b\end{matrix}\right)$

• وتكون الشحنتين التأثيريتين هما عكس (في الإشارة) الشحنتين الأصليتين.

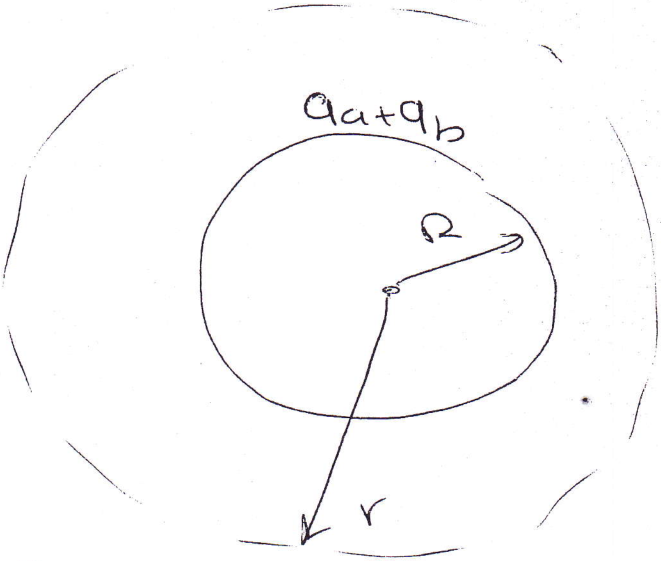
• وحيث أن الكرة متعادلة فإنه تنشأ شحنة تأثيرية أخرى ولكن على السطح الخارجي للكرة ($q_a + q_b$) لتعادل تأثير الشحنتين ($-q_a, -q_b$).

(a) $\rho_s = \dfrac{\text{Charge}}{\text{area}} \Rightarrow \rho_{sa} = \dfrac{-q_a}{4\pi a^2}, \rho_{sb} = \dfrac{-q_b}{4\pi b^2}$

$$\rightarrow \rho_{sR} = \frac{q_a + q_b}{4\pi R^2}$$

(b) $\underline{r > R}$:

Gaussian sur (sph $r > R$)



$Q_{enc} = q_a + q_b$

$\bar{D} \cdot \bar{S} = 4\pi r^2 D_r$

$$\therefore \bar{D} = \frac{q_a + q_b}{4\pi r^2} \hat{r}$$

$$\bar{E} = \frac{\bar{D}}{\epsilon} = \frac{q_a + q_b}{4\pi \epsilon r^2} \hat{r}$$

(C)

$\rightarrow$ $\underline{\text{Cavity } a}$:

$Q_{enc} = \bar{D} \cdot \bar{S}$

$q_a = 4\pi r^2 D_r$ $\quad \therefore \bar{E}_a = \dfrac{q_a}{4\pi \epsilon r^2} \hat{r}$

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$\rightarrow$ $\underline{\text{Cavity } b}$:

$Q_{enc} = \bar{D} \cdot \bar{S}$

$q_b = 4\pi r^2 D_r$

$$\bar{E}_b = \frac{q_b}{\quad} \hat{r}$$

(d) $\bar{F}_a = \bar{F}_b = 0$

لأن جسم الموصل لا يمر خلاله ال $\bar{E}$

(e)

a , c, d $\longrightarrow$ will be the same.

b $\longrightarrow$ will change.

لأن هذه الشحنة q سوف تؤثر

على قيمة المجال خارج الموصل.

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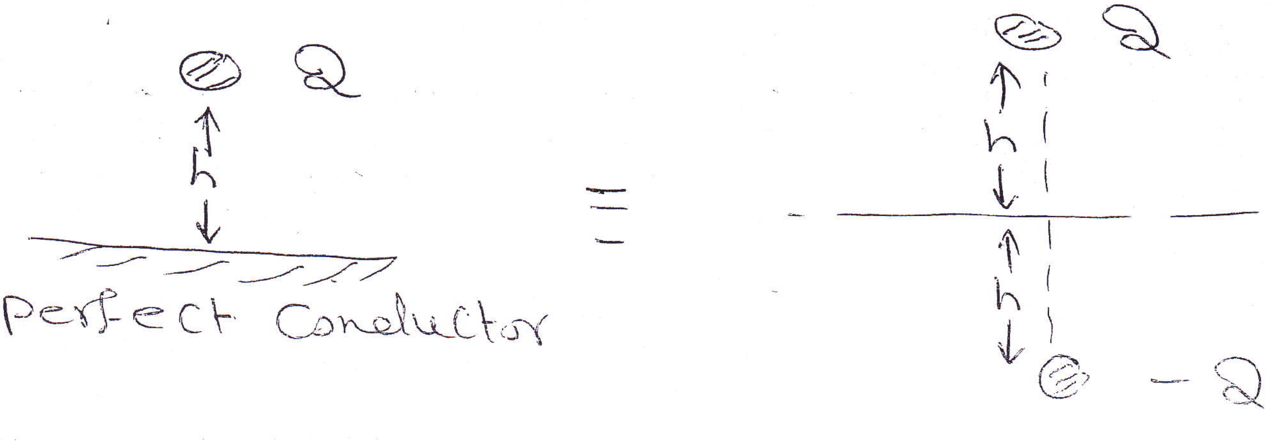
4 The method of Images:

• عند وضع شحنة كهربية $+Q$ أمام Perfect Cond.

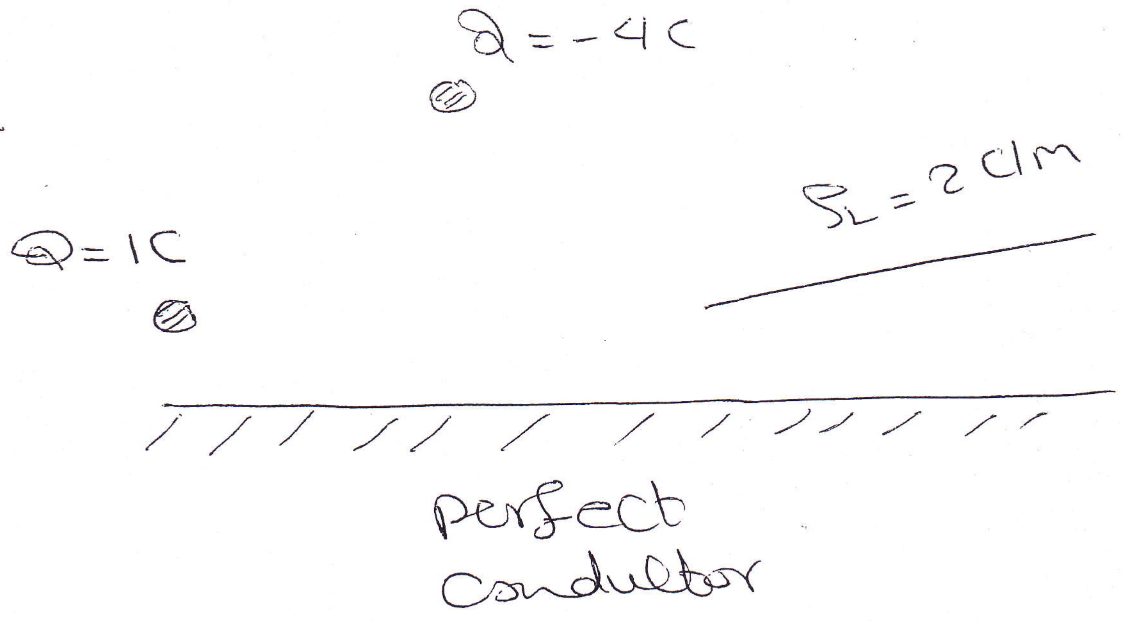
فإنها تعطي مجالاً كما لو أن هناك شحنة $-Q$ على

نفس المسافة من سطح ال Perfect Cond .

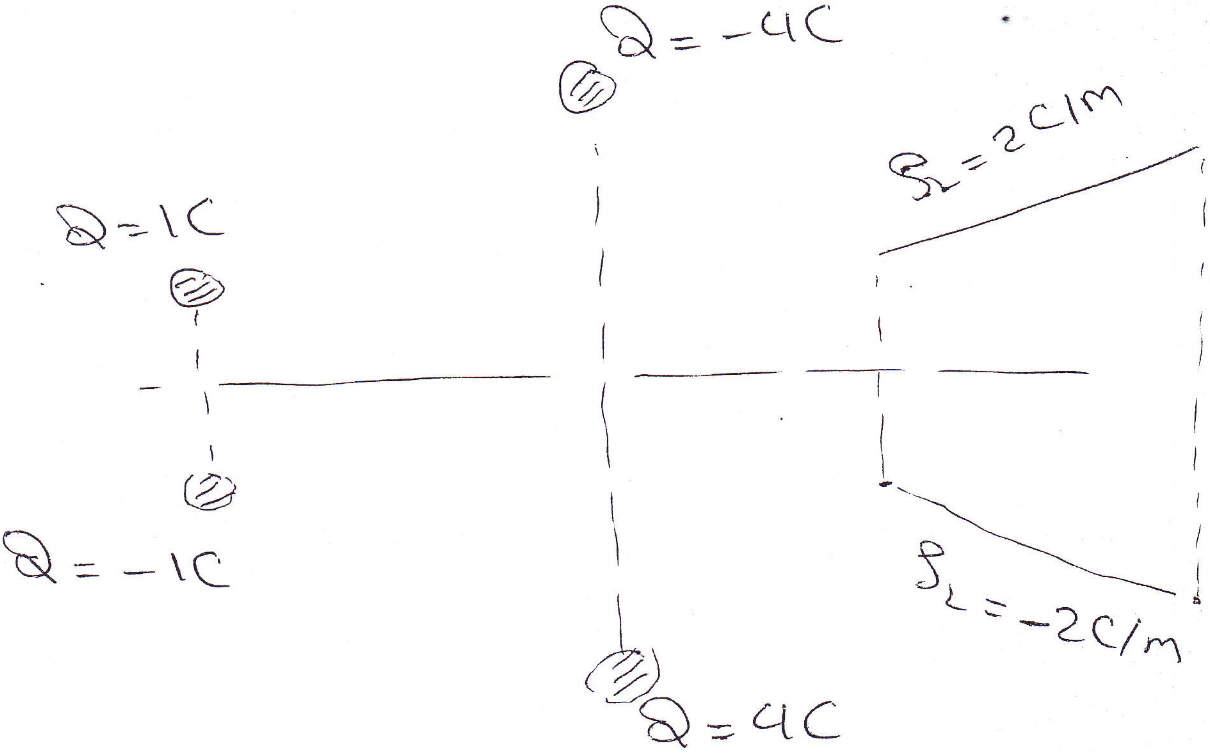


ومن هنا يمكن أن نرى أن سطح أي Perfect Cond يعمل

Ex :



→ It is equivalent to :



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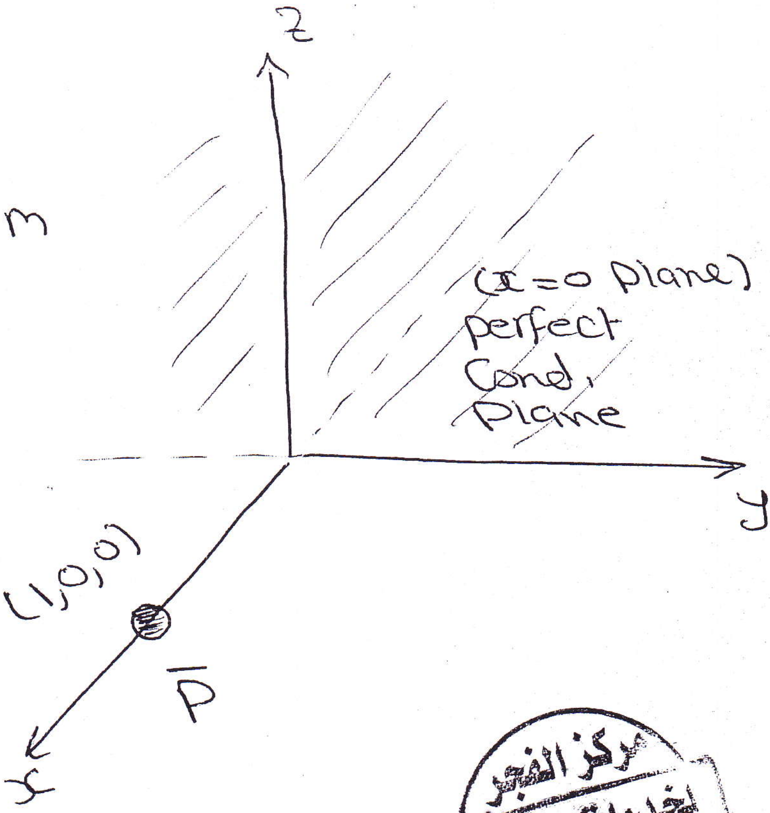
→ Dipole: $\bar{P} = 0.1\,\hat{z}\ \mu C.m$

at A(1,0,0).

→ Plane $x = 0$ is perfectly conducting.

→ find V at

P(2,0,1)



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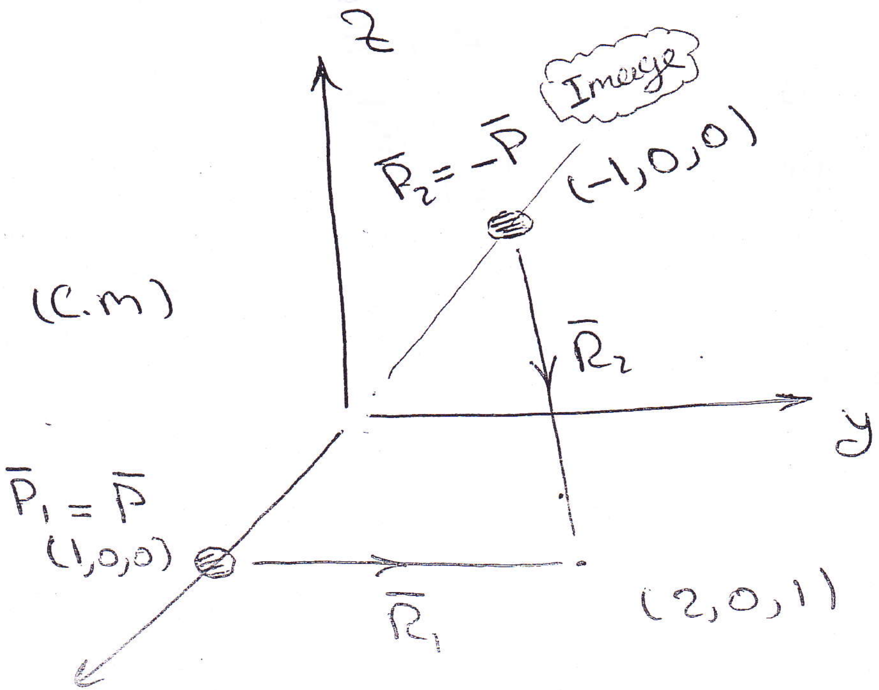
Solution

⇒ According to the Image theory:

" we can remore the perfect cond. plane and put the image of the dipole at (-1,0,0)"

$\bar{P} = 10^{-7}\,\hat{z}\ (C.m)$



Dipole potential : $V = \dfrac{\bar{P} \cdot \bar{R}}{4\pi\epsilon|R|^3}$

$$V_p = V_1 + V_2$$

$$= \frac{\bar{P}_1 \cdot \bar{R}_1}{4\pi\epsilon|R_1|^3} + \frac{\bar{P}_2 \cdot \bar{R}_2}{4\pi\epsilon|R_2|^3}$$

$\bar{R}_1 = (2,0,1) - (1,0,0) = \hat{x} + \hat{z}$

$\bar{R}_2 = (2,0,1) - (-1,0,0) = 3\hat{x} + \hat{z}$

$|R_1| = \sqrt{2}$ $\qquad$ $|R_2| = \sqrt{10}$

$$\therefore V_p = \frac{10^{-7}\hat{z} \cdot (\cancel{\hat{x}} + \hat{z})}{4\pi\epsilon(2^{3/2})} + \frac{-10^{-7}\hat{z} \cdot (\cancel{3\hat{x}} + \hat{z})}{4\pi\epsilon(10^{3/2})}$$

$$= \frac{10^{-7}}{4\pi\epsilon(2^{3/2})} - \frac{10^{-7}}{4\pi\epsilon(10^{3/2})}$$

$$= \boxed{289.34}\ V$$

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$\boxed{14}$ $\rho_l = 16 \, nC/m \longrightarrow$ line $(y=-2, z=5)$

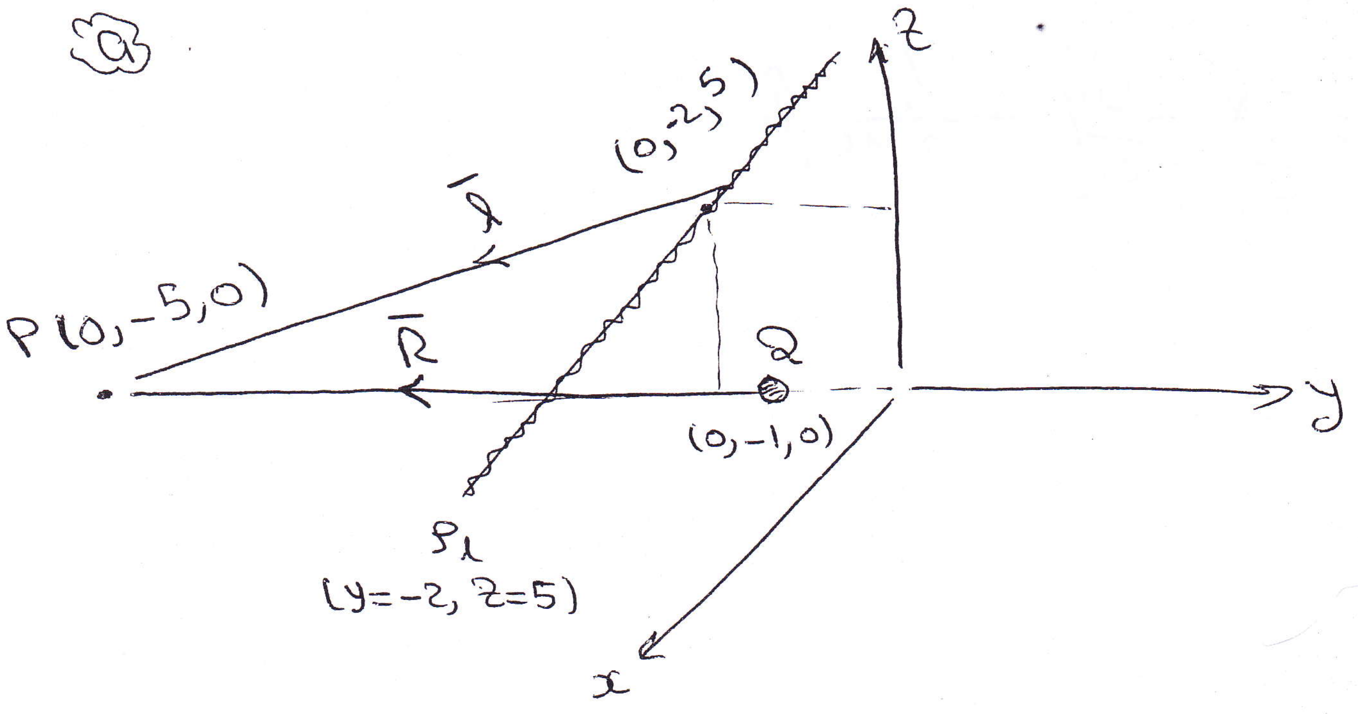
$Q = 8 \, nC \longrightarrow (0,-1,0)$

(a) find $\bar{E}\,\big|_{(0,-5,0)}$

(b) Repeat, if $y=0$ is perfectly Cond. plane.

Solution

(a)



$$\bar{E} = \bar{E}_l + \bar{E}_Q = \frac{\rho_l}{2\pi\epsilon} \cdot \frac{\bar{l}}{|l|^2} + \frac{Q}{4\pi\epsilon} \frac{\bar{R}}{|R|^3}$$

$$\bar{l} = (0,-5,0) - (0,-2,5) = -3\hat{y} - 5\hat{z}$$

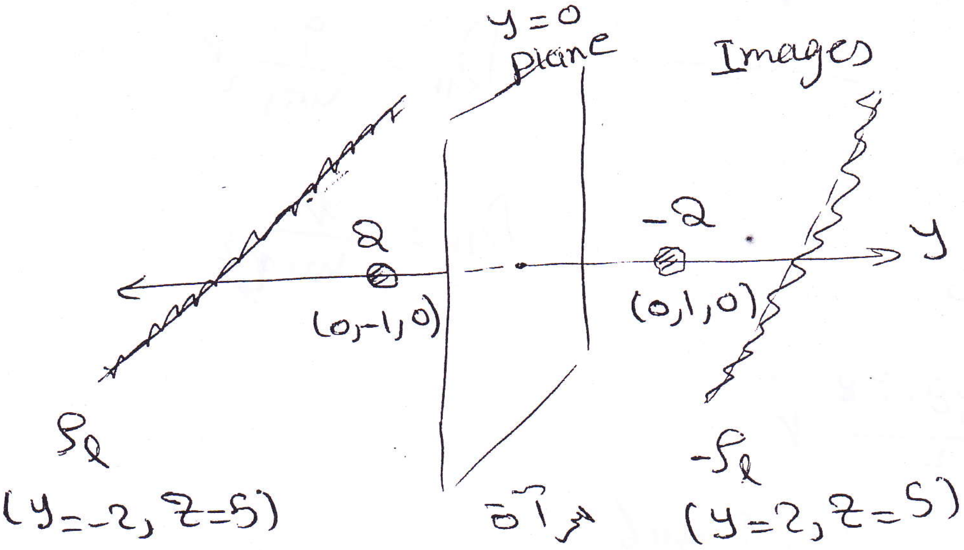
$$\bar{R} = -4\hat{y}$$

$$|l| = \sqrt{34} \qquad |R| = 4$$

$$\bar{E} = \frac{\rho_\ell}{2\pi\epsilon} \cdot \frac{-3\hat{y}-5\hat{z}}{34} + \frac{Q}{4\pi\epsilon} \cdot \frac{-4\hat{y}}{4^3}$$

$$= -29.87\,\hat{y} - 42.29\,\hat{z} \quad (V/m)$$

ⓑ Using Image Theory:



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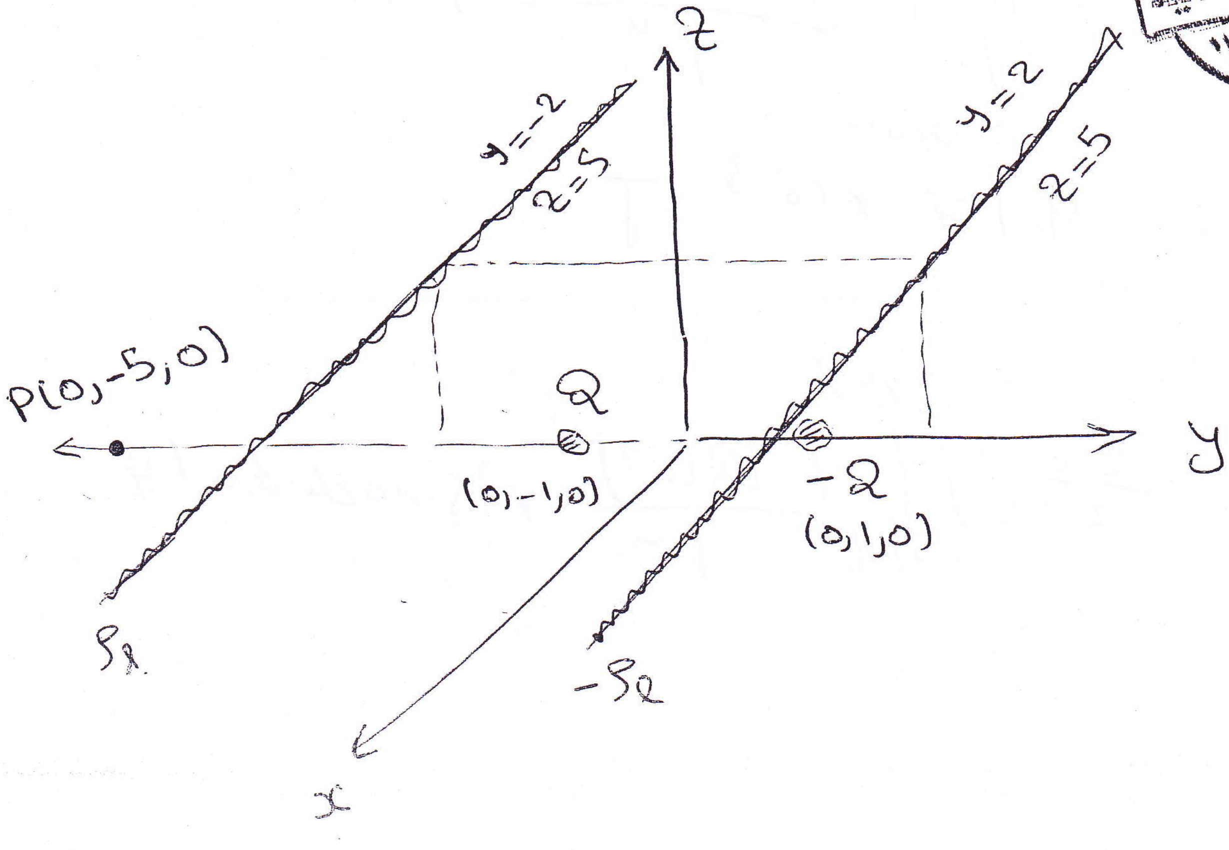

$$\bar{E}_p = \bar{E}_\ell + \bar{E}_Q + \bar{E}_I$$

$$= -29.87\,\hat{y} - 42.29\,\hat{z} + \bar{E}_I$$

$\Downarrow$

المجال الكهربي الناشئ عن الصور

line $-\rho_\ell$

point $-Q$

Images

$$\bar{\ell} = (0,-5,0) - (0,2,5) = -7\hat{y} - 5\hat{z}$$

$$|\ell| = \sqrt{74}$$

$$\bar{R} = (0,-5,0) - (0,1,0) = -6\hat{y}$$

$$|R| = 6$$

$$\bar{E}_I = \frac{-\rho_\ell}{2\pi\epsilon} \cdot \frac{-7\hat{y} - 5\hat{z}}{74} + \frac{-Q}{4\pi\epsilon} \cdot \frac{-6\hat{y}}{6^3}$$

$$= 15.6\,\hat{y} + 1.66\,\hat{z}$$

$$\therefore \boxed{\bar{E}_p = -14.27\,\hat{y} - 40.63\,\hat{z}} \ (V/m)$$

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Mansoura University
Faculty of Engineering
Communication Engineering Department

Second Year Communication
Electromagnetic Fields
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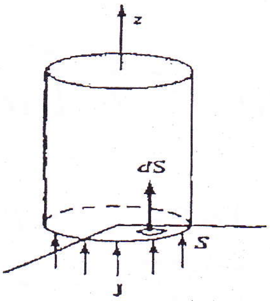

Fig. 1

[1] Find the current in the circular wire shown in Fig.1 if the current density is $\mathbf{J} = 15(1 - e^{-1000\rho})\ \mathbf{a}_z$ A/m^2. The radius of the wire is 2 mm.

[2] Two spherical cavities, of radii a and b, are hollowed out from the interior of a neutral perfectly conducting solid sphere of radius R. At the center of each cavity a point charge is placed; call these charges q_a and q_b.

(a) Find the ρ_s on the inner surfaces of the cavities and the outer surface of the sphere.

(b) What is the field outside the conductor?

(c) What is the field within each cavity?

(d) What is the force on q_a and q_b?

(e) Which of these answers would change if a third charge q is brought near the conductor?

[3] A dipole with moment $\mathbf{P} = 0.1\ \mathbf{a}_z$ μC.m is located at A (1, 0, 0) in free space and the plane x = 0 is perfectly conducting. Find the potential at the point p (2, 0, 1).

[4] A uniform line charge of 16nC/m is located along the line defined by y = −2, z = 5, while a point charge of 8nC are located at (0, -1, 0).

(a) Find **E** at P (0, -5, 0).

(b) If the plane y = 0 is perfectly conducting plane. Repeat (a).

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